Wound Core Power Transformer Design: Classical Methodology and Advanced Magnetic Field Analysis Techniques

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Abstract — **The present paper describes an overview of the classical design methodology of three phase, wound core power transformers. Moreover, advanced numerical techniques, based on the two-dimensional (2D) and three-dimensional (3D) finite element method (FEM), have been applied for the calculation of the power transformer equivalent circuit parameters (leakage inductance, short-circuit impedance). The generalization of the 3D FEM results proves its enhanced accuracy in the prediction of the wound core transformer characteristics. Experimental validation of the proposed methodology is also provided, along with the presentation of the benefits resulting from the adoption of numerical techniques in the design process.**

Keywords — **wound core transformers, design methodology, short-circuit impedance, finite element methods.**

I. INTRODUCTION

Transformers are electric machines that enable the transmission and distribution of electric energy in a simple and cost-effective way, since their efficiency overcomes 97%. The modern industry requirements necessitate the construction of a great variety of transformers that do not fit into standardized large-scale constructions. In such cases, experiential ways of electric characteristics calculation do not afford satisfying accuracy, as they concern particular geometries. Moreover, the limited delivery time does not allow the experimental verification of the predicted transformer characteristics [1, 2].

The systematic increase of computer efficiency along with the evolution of numerical methods of magnetic field simulation enable the detailed transformer magnetic field analysis with the use of low cost computational systems. The finite element method is one of the numerical methods that have prevailed in the field analysis of two and three-dimensional configurations that comprise materials with non-linear characteristics, like transformers, and may be applied within reasonable time in an appropriate personal computer.

Techniques based on finite elements are widely encountered in the technical literature. The leakage field evaluation has been extensively analysed, [3-7], as well as stray and eddy current, iron lamination characteristics and design considerations [8-13].

In the present paper, finite element techniques are used for the magnetic field analysis of three phase, wound core, power transformers under short-circuit. The analysis focuses in the leakage field evaluation and the short-circuit impedance prediction. The 3D FEM results compare favourable to the measured characteristics, with regard to the results of the classical design methodology. Therefore, in cases where a difficulty of the transformer parameters evaluation through the existing design methodology arises, it can be overcome by incorporating this magnetic field analysis technique to the existing approximating methods used by manufacturers. Furthermore, the accurate prediction of the transformer characteristics can result to its cost reduction, since the short-circuit impedance value is critical for the choice of dimensions that ensure its durability under short-circuit conditions.

The paper is organised as follows: Section II describes the geometrical characteristics of the examined wound core transformers, while Section III provides an overview of the existing transformer design methodology. In Section IV, transformer modelling with the use of 2D and 3D FEM models is presented, along with a comparison of their results to the classical design methodology and measurements. Finally, Section V concludes the paper.

II. THREE-PHASE, WOUND CORE, POWER TRANSFORMER CONFIGURATION

Fig.1 shows the active part of the three-phase, wound core, distribution transformer considered. Its magnetic circuit is of shell type and is assembled from two small and two large wound iron cores. The Low Voltage (LV) winding (secondary winding) comprises layers of copper sheet, while the High Voltage (HV) winding (primary winding) consists of copper wire (Fig. 2).

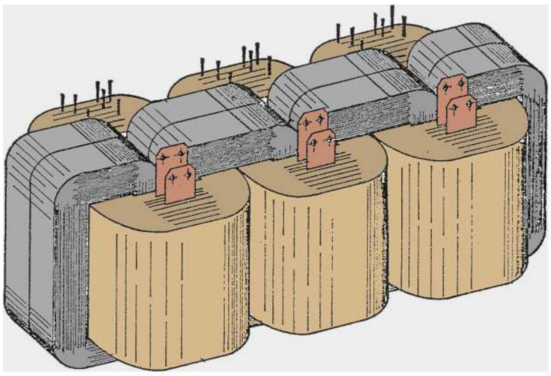

Figure 1. Active part configuration of the three-phase wound core distribution transformer considered.

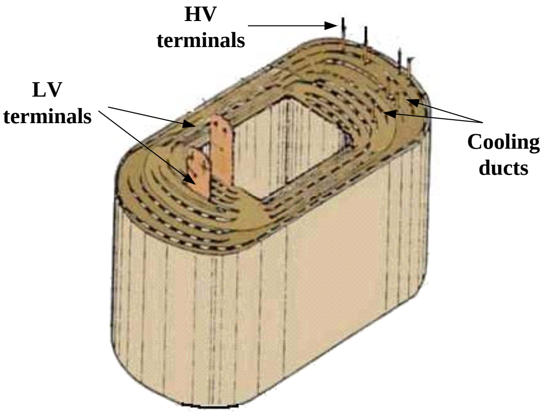

Figure 2. Perspective view of LV and HV winding of one phase.

III. CLASSICAL TRANSFORMER DESIGN METHODOLOGY

In the transformer study phase, the design engineer must satisfy the customer need for one specific transformer type, producing a design that fulfills the customer requirements and the international technical specifications [14]. For the implementation of the transformer study phase, the manufacturing industry has developed a computer software, using an optimal solution searching algorithm. The design engineer introduces the data in the computer program and the program calculates whether acceptable solutions can derive from the specified data.

During the development of the software, special consideration has been given to the data input procedure. More specifically, there are seven groups of variables for the design of a three-phase distribution transformer [15]:

- *Description variables* (e.g., rated power, rated LV and HV, frequency, material of LV and HV coil, LV and HV connection, …)
- *Variables that rarely change* (e.g., LV and HV BIL, core space factor, turns direction space factor, short-circuit factor, …)
- *Variables with default values* (e.g., LV and HV taps, guarantee and tolerance fields for load loss, no-load loss and impedance, …)
- *Cost variables* (e.g., cost per weight unit for LV and HV conductor, magnetic steel, oil, insulating paper, duct strips, corrugated panels, …)
- *Optional variables* (e.g., variables that either can be calculated by the program or defined by the user)
- *Various parameters* (e.g., type of LV and HV conductor, number of LV and HV ducts, LV and HV maximum gradient, maximum ambient temperature, maximum winding temperature, …)
- *Variables for conductor cross-section calculations* (LV and HV conductor cross-sections can be defined by the user or can be calculated using current density, or thermal short-circuit test)
- Solution loop variables (e.g., LV turns, dimensions of core, magnetic induction, LV and HV cross-section)

Using this program, and giving enough alternative values to the loop variables, enough candidate solutions are made. For each one of the candidate solutions, it is checked if all the specifications (limits) are satisfied, and if they are satisfied, the cost is calculated and the solution is characterized as acceptable. On the other hand, the candidate solutions that violate the specifications are characterized as non-acceptable solutions. Finally, from the acceptable solutions, the transformer with the minimum cost is selected which is the optimum technical and economical transformer.

Giving n_{LV} different values for the turns of the low voltage coil, n_D values for the core's dimension D (width of core leg), n_{FD} tries for the magnetic flux density, n_G different values for the core's dimension G (height of core window), cs_{LV} different values for the calculation of the cross-section area of the low voltage coil and cs_{HV} different values for the calculation of the cross-section of the high voltage coil, the total candidate solutions (loops of the computer program), n_{loops}, are calculated from the following equation:

$$n_{loops} = n_{LV} * n_D * n_{FD} * n_G * cs_{LV} * cs_{HV} \qquad (1)$$

The solution algorithm of the technical and economical optimum transformer is presented in Figure 3.

During the algorithm iterations, the transformer short-circuit impedance calculation is based on an approximating methodology for the evaluation of the windings leakage inductance. According to this methodology, the transformer is represented by an equivalent two-dimensional geometry, based on cylindrical symmetry, which takes into account the contribution of the winding parts outside the core window (i.e. in the third dimension) with the use of the Rogowski coefficients (modified transformer geometry).

The total leakage inductance (L_{leak}), resulting the above analysis, is used for the calculation of the inductive voltage drop (IX), through equation (2).

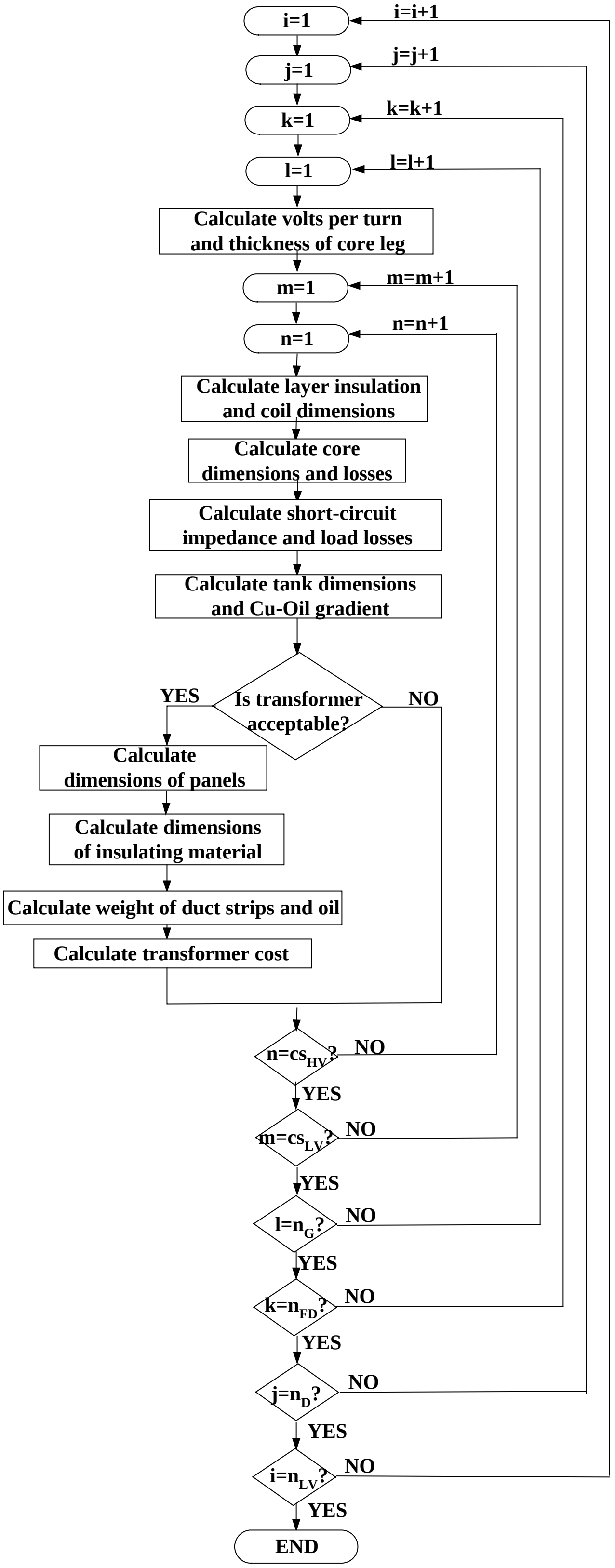

Figure 3. Solution algorithm of the technical and economical optimum transformer.

$$IX(\%) = \frac{I_{LV} \cdot 2 \cdot \pi \cdot f \cdot N_{LV}^2 \cdot L_{leak}}{V_{LV}} \qquad (2)$$

where,

I_{LV} : LV winding nominal current
V_{LV} : LV winding nominal voltage
f : frequency
N_{LV} : LV winding turns

Finally, the short-circuit impedance U_k is given by:

$$U_k = \sqrt{IX^2 + IR^2} \qquad (3)$$

where IR is the windings resistive voltage drop, calculated with the use of the theoretical value of the transformer load losses.

IV. TRANSFORMER MODELLING WITH THE FINITE ELEMENT METHOD

The finite element method is a numerical technique for the solution of problems described by partial differential equations. The considered field is represented by a group of finite elements. The space discretization is realized by triangles or tetrahedra if the problem is two or three-dimensional respectively. Therefore, a continuous physical problem is converted into a discrete problem of finite elements with unknown field values in their vertices nodes. The solution of such a problem reduces into a system of algebraic equations and the field values inside the elements can be retrieved with the use of calculated values in their indices. Therefore, the solution of the 2D or 3D magnetostatic problem describing the transformer field reduces into the calculation of the magnetic field density at each vertex node of the triangles or tetrahedra of its 2D or 3D mesh, respectively.

A. Two-dimensional Finite Element Method

Figure 4 shows the equivalent two-dimensional finite element model, representing the transformer active part. As in the case of the classical design methodology, the contribution of the winding parts outside the core window (which cannot be represented in the xy-plane), is taken into account by a proper modification of the winding height, with the use of the Rogowski coefficients (modified transformer geometry).

The 2D triangular mesh used in the finite element calculations is shown in Fig. 5, consisting of approximately 20000 nodes. The density of the mesh is greater in the windings area, which is crucial for the leakage inductance evaluation under short-circuit test. Moreover, special consideration has been given to the homogeneity of the overall mesh, as it consists an important factor for the accuracy of the finite element calculations.

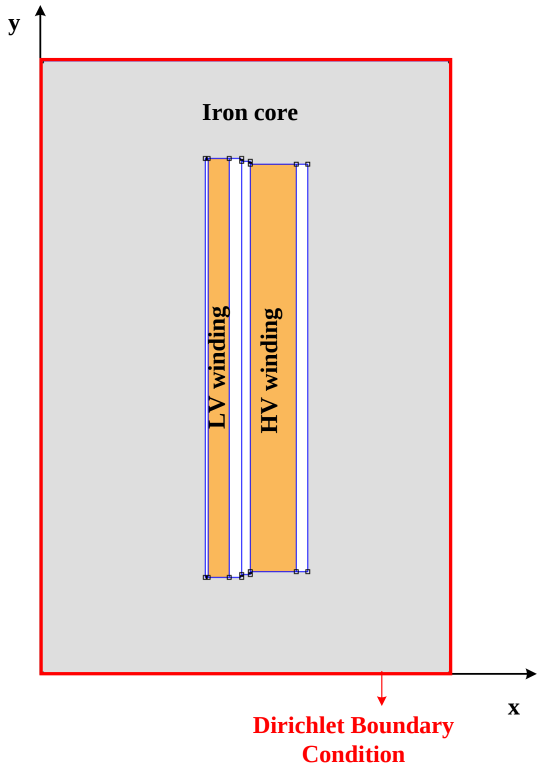

Figure 4. 2D FEM model of the transformer active part (modified geometry).

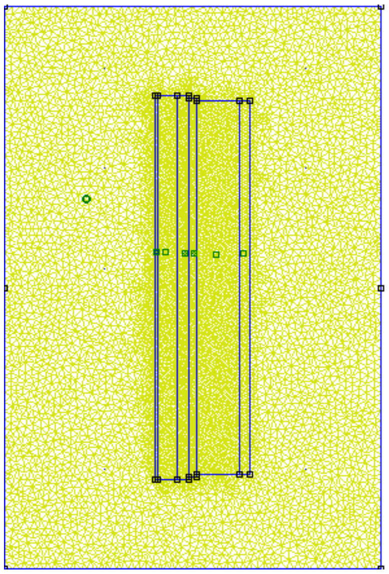

Figure 5. 2D FEM mesh of the transformer active part.

Figure 6 illustrates the magnetic flux density magnitude distribution during short-circuit test for a 1250 kVA, 20/0.4 kV wound core distribution transformer. The total leakage inductance (L_{leak}) is equal to $1.637 \cdot 10^{-7}$ H. The resulting inductive voltage drop (IX) is calculated with the use of (2). The value of IX calculated as described above is equal to 5.78%, while the value calculated by the classical design methodology is equal to 5.8%. The results are very close, due to the similar geometry approximations in the 2D representation of the transformer configuration, adopted by both methods.

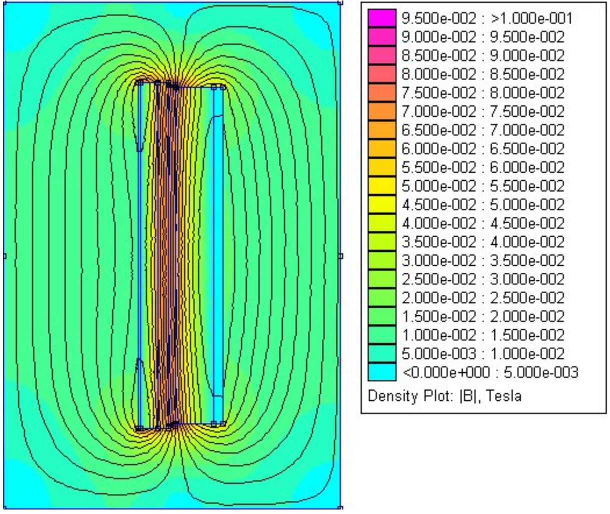

Figure 6. Variation of the magnetic induction magnitude under short-circuit test for a 1250 kVA wound core transformer (2D FEM).

B. Three-dimensional Finite Element Method

In the 2D FEM analysis, the magnetic field calculation was conducted with the use of the magnetic vector potential $\vec{A}$. However, in the case of 3D problems, the use of the vector potential results to great complexity, due to the large number of unknowns. Therefore, the use of magnetic scalar potential Φ_m is preferred in the case of 3D magnetostatic problem solution.

In most of the developed scalar potential formulations this calculation of Φ_m is realized with the use of the following equation:

$$\vec{H} = \vec{H}_s - \nabla\Phi_m \tag{4}$$

where H_s is the source field density given by Biot-Savart's Law:

$$\vec{H}_s = \frac{1}{4\pi}\int_V \frac{\vec{J}\times(\vec{r}-\vec{r}')}{\left|\vec{r}-\vec{r}'\right|^3}dV \tag{5}$$

However, the above calculation presents the drawback of considerable computational effort, due to the prior source field calculation with the use of (4). In the present paper, a particular scalar potential formulation is adopted [16, 17]: according to this method, the magnetic field strength H is conveniently partitioned to a rotational and an irrotational part as follows:

$$\vec{H} = \vec{K} - \nabla\Phi_m \tag{6}$$

where Φ_m is the scalar potential extended all over the solution domain while $\vec{K}$ is a vector quantity (fictitious field distribution), defined in a simply connected subdomain comprising the conductor, that satisfies Ampere's law and is perpendicular on the subdomain boundary.

Figure 7 illustrates the perspective view of the three-dimensional FEM model of the transformer active part, comprising the iron core, low and high voltage windings of one phase.

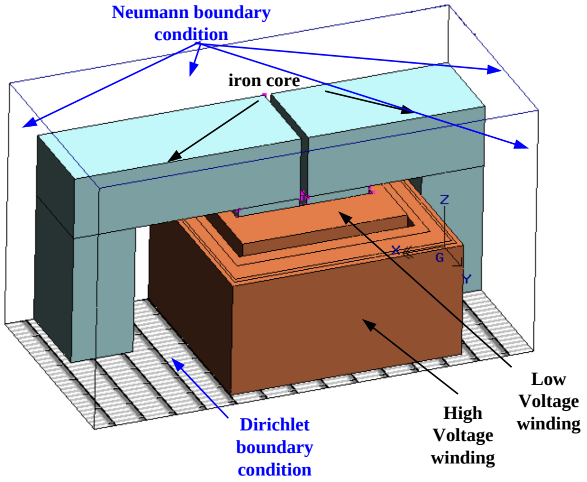

Figure 7. 3D FEM model of the transformer active part.

Due to the symmetries of the problem, the solution domain was reduced to one fourth of the device (although there is a slight disymmetry due to the terminal connections in one side). These symmetries were taken into account by the imposition of Dirichlet boundary condition ($\Phi=0$) along xy-plane and Neumann boundary condition ($\frac{\partial \Phi}{\partial n}=0$) along the other three faces of the air box that surrounds the transformer active part.

The 3D FEM mesh of the transformer active part, consisting of approximately 90000 nodes, is shown in Fig. 8, while Figure 9 illustrates the magnetic flux density magnitude distribution during short-circuit test for a 630 kVA, 20-15/0.4 kV wound core distribution transformer.

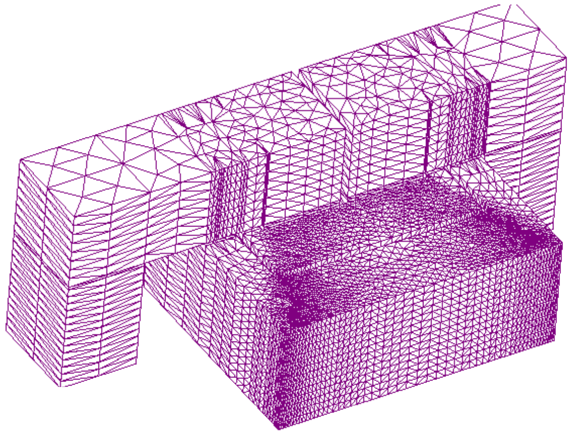

Figure 8. Three-dimensional finite element mesh of the transformer active part.

The 3D FEM results were used to calculate the transformer short circuit impedance. The total windings leakage inductance L_{leak} was calculated with the use of the magnetic energy of the model (W_m), according to (7):

$$L_{leak} = \frac{2 \cdot W_m}{(NI)^2} \qquad (7)$$

where NI are the ampere-turns of the HV or LV winding (they are considered equal in the case of short circuit). The inductive voltage drop IX(%), as stated in Section III, derives from (2), while the transformer short circuit impedance (Uk) results from (3).

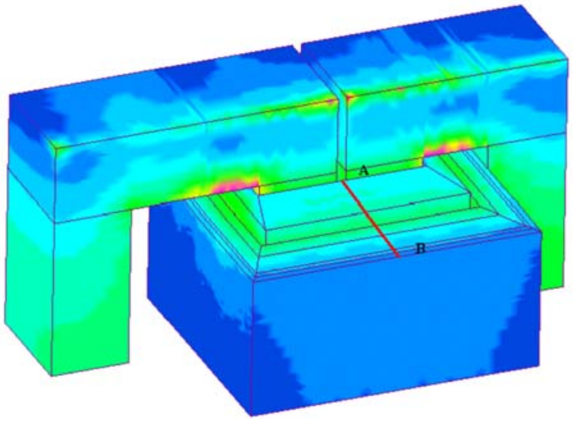

Figure 9. Variation of the magnetic induction magnitude under short-circuit test for a 630 kVA wound core transformer (3D FEM).

The short-circuit impedance value calculated with the use of the 3D FEM model in the case of the 630 kVA transformer was equal to 3.83%. The classical design methodology resulted to U_k value equal to 4%, while the measured short-circuit impedance of the constructed transformer was found equal to 3.80%. Therefore, the deviation between 3D FEM and the measured value is equal to 0.8%, while the deviation of the classical method is equal to 5.3%. This difference demonstrates the ability of 3D FEM to accurately predict the transformer characteristics, due to the better representation of the real transformer geometry. On the other hand, the classical design methodology adopts geometry approximations that result to greater deviation from the actual (measured) short-circuit impedance values. The accuracy of 3D FEM has been validated by application of the method during the design of transformers with different power ratings and voltage levels and comparison to the measured short-circuit impedance value of the constructed transformers.

V. CONCLUSIONS

In the present paper, the classical design methodology as well as 2D and 3D FEM models have been applied for the calculation of the leakage field and the short-circuit impedance of three-phase, wound core, power transformers. The results of the method were compared to the ones of the actual (measured) values, in several cases of wound core transformers. The results of 2D FEM are close to the ones of the existing methodology, as both methods adopt the same approximations for the two-dimensional representation of the transformer geometry. The 3D FEM model provides higher accuracy in the prediction of the short-circuit impedance, with respect to the measured value, due to the better representation of the transformer geometry. Therefore, the adoption of this numerical technique during the design phase is able to enhance the manufacturer's ability to predict the transformer operational characteristics, thus resulting to better performance and cost reduction.

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